Nuclear Disintegration in Magnetic Fields

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Abstract

We employ the Weizsäcker-Williams method of virtual quanta to study disintegration of nuclei in magnetic field. We explore a variety of field configurations and conclude that for the energy range of interest for applications to cosmic rays ($10^{18} - 10^{21}$ eV) such disintegrations are not a significant source of energy or flux loss for any realistic acceleration mechanism.

1 Introduction

Understanding the interactions of cosmic rays with cosmic backgrounds is essential for solving the puzzle of ultrahigh-energy cosmic rays [1], as well as for understanding the most powerful natural accelerators, such as active galactic nuclei, gamma-ray bursts, *etc.* Since their discovery, cosmic rays with energies beyond the Greisen-Zatsepin-Kuzmin cutoff [2], $E > 10^{19.6}$eV, have presented astrophysics with several important and unresolved problems. The composition of ultrahigh-energy cosmic rays remains unknown. In particular, they may be protons, photons, or nuclei. In the latter case, interactions with the background photons cause nuclear disintagration [3].

A number of possible acceleration mechanisms have been proposed. Some of them, such as neutron stars [4], have very large magnetic fields. To a highly relativistic particle, such magnetic fields are much larger and they are accompanied by a matching electric field. Thus, the magnetic field acts like an intense photon field. It is therefore natural to ask whether such magnetic fields may cause energy losses in relativistic nuclei, through photodisintegration.

In this paper we explore the possibility that the static magnetic fields of an accelerator may cause photodisintegration of relativistic nuclei. We will use the Weizsäcker-Williams method of virtual quanta, as described in reference [5]. The same method has been used to study photodisintegration of nuclei in the Coulomb elctric fields of target nuclei [6]. In sections 2-4 we set up the method by first transforming the magnetic field to the rest frame of the accelerated particle, second, decomposing this field as a collection of virtual photons, and third, studying the effect of this spectrum of virtual photons on the propagating nuclei. In section 5 we use our method to find the effective photon spectrum of a variety of magnetic fields. In section 6 we use some simple cross section data to place limits on the required magnetic fields to get significant photodisintegration. In section 7 we generalize the method to apply to accelerating particles. Finally in section 8 we conclude.

2 Transformation of the Magnetic Field

If we attempt to study the disintegration of a relativistic nucleus in a static magnetic field from the rest frame of the field, we run into a number of problems. In this frame the field does not look like a collection of quasi-real photons and therefore the experimental data on photodisintegration would be useless to us. We would then have to resort to a full field theoretic calculation of the nuclear disintegration. Such theory is both complicated and incomplete. Therefore, this path is not very helpful.

Instead we will begin by transforming our magnetic field to the rest frame of the nucleus. In this frame, the field looks very much like a collection of real photons (hence the term quasi-real). We express the field as a collection of quasi-real photons and use the known experimental data on photodisintegration to calculate the likelihood that the nucleus will disintegrate in the magnetic field.

We begin with a static magnetic field $\mathbf{B}(x, y, z)$. For simplicity we will Lorentz boost this in the x direction, so that the argument becomes $x \to \gamma(x + vt)$, $y \to y$, $z \to z$. The transformations of the fields are also simple and can be found in any standard text such as [7].

$$
\begin{aligned}
\mathbf{E}'_\perp &= \gamma\left(\mathbf{E}_\perp + \frac{\mathbf{v}}{c}\times\mathbf{B}_\perp\right) && (1)\\
\mathbf{E}'_\parallel &= \mathbf{E}_\parallel && (2)\\
\mathbf{B}'_\perp &= \gamma\left(\mathbf{B}_\perp - \frac{\mathbf{v}}{c}\times\mathbf{E}_\perp\right) && (3)\\
\mathbf{B}'_\parallel &= \mathbf{B}_\parallel && (4)
\end{aligned}
$$

We now substitute this expression into our original form for the magnetic field and find

$$
\begin{aligned}
\mathbf{B}' &= \gamma\mathbf{B}_\perp + \mathbf{B}_\parallel && (5)\\
\mathbf{E}' &= \gamma\frac{\mathbf{v}}{c}\times\mathbf{B}. && (6)
\end{aligned}
$$

Here we note two important aspects of this electromagnetic field. First, the parallel component of the magnetic field is irrelevant. It has no factor of γ enhancement, and it has no partner electric field and therefore has no effect on a particle at rest. This is expected as magnetic fields along the direction of a charged particle's motion generally have little or no effect.

Next, and more importantly, we notice that the electric field and the perpendicular component of the magnetic field are perpendicular to each other and, for ultrarelativistic particles ($v \approx c$), approximately the same magnitude. Therefore, they can be viewed as a superposition of quasi-real photons. This is the subject of our next section.

3 The Power Spectrum

We now proceed to calculating the power and number spectrum of the quasi-real photons. Using eq. (15.51) of Ref. [5], we find that

$$\frac{dI}{d\omega} = \frac{c}{2\pi}|\mathbf{E}(\omega)|^2. \tag{7}$$

Where $\mathbf{E}(\omega)$ is the Fourier transform of the electric field defined by

$$\mathbf{E}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathbf{E}(t)e^{i\omega t}\,dt. \tag{8}$$

Substituting in equation 6 for $\mathbf{E}(t)$ and changing variables we find

$$\begin{aligned}\mathbf{E}'(\omega, x, y, z) &= \int_{-\infty}^{\infty} \beta \times \mathbf{B}(\gamma(x+\beta ct), y, z) e^{i\omega t} dt \\ &= \frac{1}{\beta c\sqrt{2\pi}} e^{i\frac{\omega x}{\beta c}} \int_{-\infty}^{\infty} \beta \times \mathbf{B}(x', y, z) e^{i\frac{\omega x'}{\gamma\beta c}} dx'. \end{aligned} \tag{9}$$

Where $\beta = \frac{\mathbf{v}}{c}$.

This equation was derived with the assumption that the particle follows a straight path through the magnetic field, but it is easy to generalize it. The integral along x will simply become a path integral along the classical trajectory of the particle $\mathbf{r}(\ell)$, where ℓ is the path length. Then, droping factors of β and the irrelevant phase factor in front of the integral, we get

$$\mathbf{E}(\omega) = \frac{1}{c\sqrt{2\pi}} \int_{\text{path}} \beta \times \mathbf{B}(\mathbf{r}(\ell)) e^{i\frac{\omega}{\gamma c}\ell} d\ell. \tag{10}$$

4 Photon flux and disintegration probability

It is now straightforward to calculate the disintegration probability for the nucleus. First, we relate the intensity to the flux of photons using

$$\frac{dI}{d\omega} = \hbar\omega \frac{dn}{d\omega}.$$

Theerefore,

$$\frac{dn}{d\omega} = \frac{1}{\hbar\omega} \frac{c}{2\pi} |\mathbf{E}(\omega)|^2.$$

Next we multiply this flux by the experimentally determined cross section for photodisintegration of the nucleus to find the differential decay probability.

$$dP_{\text{decay}} = \sigma_\gamma(\omega) dn(\omega) = \sigma_\gamma(\omega) \frac{1}{\hbar\omega} \frac{c}{2\pi} |\mathbf{E}(\omega)|^2 d\omega. \tag{11}$$

One change should be made at this point. So far we have neglected the possibility that the particle has already decayed. Thus, if we integrate equation (11) we may get answers greater than 1, which is impossible. To rectify this we need to multiply the right hand side of the equation by the probability that the particle has not disintegrated. Thus, equation (11) would be better written as

$$dP_{\text{decay}} = -dP_{\text{survive}} = P_{\text{survive}} \sigma_\gamma(\omega) dn(\omega).$$

This can then be integrated to find that in fact the correct relation is

$$\ln P_{\text{survive}} = -\int_0^\infty \sigma_\gamma(\omega) dn(\omega) = -\int_0^\infty \sigma_\gamma(\omega) \frac{1}{\hbar\omega} \frac{c}{2\pi} |\mathbf{E}(\omega)|^2 d\omega. \tag{12}$$

5 Various field configurations

We now examine a variety of field configurations to see how $\mathbf{E}(\omega)$ depends on $\mathbf{B}(t)$. We define $k = \frac{\omega}{\gamma c}$.

5.1 Gausian B-field

Let us suppose that the field is of the form $\mathbf{B} = \hat{j} B_0 \exp\left\{-\frac{x^2}{a^2}\right\}$ and the nucleus is traveling at ultrareletivistic speed in the x direction. From equation (10) we find that

$$E(\omega) = \hat{k} \frac{B_0}{c\sqrt{2}} a \exp\left\{-\frac{\omega^2 a^2}{4\gamma^2 c^2}\right\} \tag{13}$$

We note that for sufficiently small values of a this is relatively flat, but that, for large values of a, $E(\omega)$ falls off very quickly with increasing ω. However, for large enough values of gamma even very large values of ω may have noticable contributions.

5.2 Constant B-field with various cutoffs

Next we consider a B-field that is relatively constant over a large region of size a and which falls to 0 over a much smaller region of size b. The method we will use is to convolve a constant magnetic field with some approximation to a delta funtion, thus introducing a smoother cutoff. This provides a nice way to compare different kinds of cutoffs on the same distance scale. Also the Fourier transforms are then fairly easy to perform because the Fourier transform of a convolution is just the product of the Fourier transforms. For the rest of this subsection we will leave off the factor of $1/c\sqrt{2\pi}$ in equation (10).

5.2.1 Constant Magnetic Field

We'll start with a constant magnetic field:

$$\mathbf{B_0}(x) = \begin{cases} \mathbf{B}_0 & x \in (-a/2, a/2) \\ 0 & \text{otherwise.} \end{cases}$$

The Fourier transform of this is simply

$$\mathbf{B}(k) = \frac{\mathbf{B}_0}{k}\, 2\sin\left(\frac{ka}{2}\right).$$

If $ka \ll 2\pi$ then this is nearly constant, however I expect that is unlikely. In the opposite limit $ka \gg 2\pi$ the oscilations in the sine function are enough that we can just replace it by it's root mean square value of $\sqrt{2}$.

It is worth noting that for this configuration the B-field itself is discontinuous and the field falls off as $1/k$ for large values of k.

5.2.2 Square pulse δ-function

From now on we will modify $\mathbf{B}$ by convolving it with some function $g(x, b)$ that has a characteristic width of b and becomes a δ-function in the limit that $b \to 0$. (Basically this is just requiring that the area be normalized to 1.) If g were a true δ- function this would just give the same form we found before. To state our assumptions mathematically

$$\mathbf{B}(x) = \int_{-\infty}^{\infty} g(x - x')\mathbf{B_0}(x')dx'$$

$$\int_{-\infty}^{\infty} g(x - x') = 1.$$

As our first approximate δ-function we will use a square pulse.

$$g(x) = \begin{cases} \frac{1}{b} & x \in (-b/2, b/2) \\ 0 & \text{otherwise.} \end{cases}$$

The Fourier transform is simply $\tilde{g}(k) = \frac{\sin{(kb/2)}}{(kb/2)}$ so that the Fourier transform of the B-field is

$$\mathbf{B}(k) = \frac{\mathbf{B}_0}{k} 2\sin\frac{ka}{2}\frac{\sin{(kb/2)}}{(kb/2)}. \tag{14}$$

Note that the small k ($k \ll 1/b$) behavior is essentially unchanged, but that for large values of k ($k \gg 1/b$) the field now falls off like $1/k^2$. Also note that whereas before the B-field itself was discontinuous, now only its first derivative is discontinuous.

5.2.3 Triangle pulse

The next step is to consider a continuous approximate δ-function with a discontinuous first derivative. This will make the B-field continuous through the first derivative. We find the following results.

$$g(x) = \begin{cases} \frac{1}{b}(1+\frac{x}{b}) & x \in (-b,0) \\ \frac{1}{b}(1-\frac{x}{b}) & x \in (0,b) \\ 0 & \text{otherwise.} \end{cases}$$

$$\tilde{g}(k) = \frac{2(1-\cos kb)}{k^2b^2} = \left(\frac{\sin(kb/2)}{(kb/2)}\right)^2$$

$$\mathbf{B}(k) = \frac{\mathbf{B}_0}{k} 2\sin\frac{ka}{2}\left(\frac{\sin(kb/2)}{(kb/2)}\right)^2 \tag{15}$$

Again the behavior for small kb is relatively stable, while for large kb the function falls off as $1/k^3$

5.2.4 Parabolic pulse

Now we make the approximate δ-function from parabolas joined together in such a way that the function and first derivative are both continuous. Thus the B-field will have continuous 1st and 2nd derivatives. Thus,

$$g(x) = \begin{cases} \frac{1}{b^3}(\frac{3}{4}b^2 - x^2) & x \in (-b/2, b/2) \\ \frac{1}{2b^3}(x - \frac{3}{2}b)^2 & x \in (b/2, 3b/2) \\ \frac{1}{2b^3}(x + \frac{3}{2}b)^2 & x \in (-3b/2, -b/2) \\ 0 & \text{otherwise.} \end{cases}$$

$$\tilde{g}(k) = \left(\frac{\sin(kb/2)}{kb/2}\right)^3$$

$$\mathbf{B}(k) = \frac{\mathbf{B}_0}{k} 2\sin\frac{ka}{2}\left(\frac{\sin(kb/2)}{kb/2}\right)^3. \tag{16}$$

Clearly, the observed pattern is continuing. I have chosen a very particular form for the parabolic pulse. It is the pulse derived by convoluting the square and triangular pulses, and thus gives the very simple form listed. However, other parabolic pulses of width b and unit area share the property that the Fourier transform goes smoothly to 1 for small k and falls off as

k^{-4} for large k. I am uncertain how to prove this pattern in general, and the specifics are getting increasingly more difficult so I will not attempt smoother cutoffs of this form.

5.2.5 Gaussian Pulse

It is; however, simple enough to explore the behavior of a particular perfectly smooth cutoff by using a gausian pulse as our approximate δ-function. This will yield a magnetic field that falls off quickly, but with no discontinuous derivatives. We expect to find that $\mathbf{B}(k)$ falls off more quickly than any power of k. This is exactly what we find.

$$g(x) = \frac{1}{b\sqrt{\pi}} \exp -\frac{x^2}{b^2}$$

$$\tilde{g}(k) = \exp -\frac{k^2 b^2}{4}$$

$$\mathbf{B}(k) = \frac{\mathbf{B}_0}{k} 2 \sin \frac{ka}{2} \exp -\frac{k^2 b^2}{4} \tag{17}$$

5.2.6 Summary and Discussion

Table 1 summarises $\mathbf{B}(k)$ for the various field configurations discussed in this subsection. The last collumn simplifies the form of $\mathbf{B}(k)$ by substituting a root mean square value for the oscillatory part.

We should be clear at this point about what is meant by a discontinuity. Certainly in any real situation we would expect the field and all of its derivatives to be smooth down to the atomic scale. However, from the point of view of the Fourier transform, any change that occurs over a scale that is small compared to the wavelength, $\lambda = 2\pi/k$, will have the same effect as a discontinuity. This can be seen in table 1. Each of the forms reduces to the case where the field itself is discontinuous if the smoothing takes place over a distance that is small compared to the wavelength. In other words if $kb \ll 1$ or equivalently $b \ll \lambda/2\pi$ then the smoothing does not matter and $\mathbf{B}(k) = \frac{\mathbf{B}_0}{k} 2 \sin \frac{ka}{2}$.

By virtue of the Fourier transform, the more severe the discontinuity in $\mathbf{B}(k)$, the slower the fall-off in $\mathbf{B}(k)$. If the first discontinuity in $\mathbf{B}$ or it's derivatives appears at order $\mathbf{B}^{(i)}$ or the ith derivative of $\mathbf{B}$, then for large k the $\mathbf{B}(k)$ appears to fall off as $k^{-(i+1)}$.

Table 1: $\mathbf{B}$ (k)

Smoothing Pulse	First Discontinuity	$\mathbf{B}$(k)	simplified $\mathbf{B}$(k)
True δ-function	$\mathbf{B}$	$\frac{\mathbf{B}_0}{k} 2 \sin \frac{ka}{2}$	$\frac{\sqrt{2}\mathbf{B}_0}{k}$
Square Pulse	$\mathbf{B}'$	$\frac{\mathbf{B}_0}{k} 2 \sin \frac{ka}{2} \frac{\sin(kb/2)}{(kb/2)}$	$\frac{2\mathbf{B}_0}{k^2 b}$
Triangle Pulse	$\mathbf{B}''$	$\frac{\mathbf{B}_0}{k} 2 \sin \frac{ka}{2} \left(\frac{\sin(kb/2)}{(kb/2)}\right)^2$	$\frac{2\sqrt{3}\mathbf{B}_0}{k^3 b^2}$
Parabola Pulse	$\mathbf{B}'''$	$\frac{\mathbf{B}_0}{k} 2 \sin \frac{ka}{2} \left(\frac{\sin(kb/2)}{(kb/2)}\right)^3$	$\frac{2\sqrt{10}\mathbf{B}_0}{k^4 b^3}$
Gaussian Pulse	None	$\frac{\mathbf{B}_0}{k} 2 \sin \frac{ka}{2} \exp -\frac{k^2 b^2}{4}$	$\frac{\sqrt{2}\mathbf{B}_0}{k} \exp -\frac{k^2 b^2}{4}$

The pattern that we see here appears to be a general property of the Fourier transform that we are taking. The Fourier transform is most sensitive to structure (changes in magnetic field, discontinuities in the field or its derivatives, etc.) that are on the same scale or smaller than the wavelength. The smoother the field is on this length scale, the more heavily surpressed the Fourier transform is. This means that in order to see a significant effect on nuclei, the magnetic field will need to have significant structure on the scale $\gamma\lambda_{dis}$ where λ_{dis} is a typical wavelength for photo- disintegration. As we shall see later, for the highest known cosmic-ray energies, this requires structure on the order of a few milimeters. However, the length scale increases with increasing cosmic ray energy. Also, repeating variations in magnetic field, which would result in a smaller average field for other measurements, would increase the size of $\mathbf{B}(k)$

5.3 Dipole Field

A dipole field can easily be represented by a scalar potential. This is a bit unconventional for a magnetic field, but it can be done, so long as there are no currents. (This way $\nabla \times \mathbf{B} = 0$.) The field $\mathbf{B}$ is then simply $\mathbf{B} = \nabla\phi^*$. We can now do the Fourier transform on the single potential rather than on the three components of the field. To justify this, one can easily show that if

$$B(x, y, z) = \nabla\phi^*(x, y, z),$$

then

$$B_{x,y}(x, y, k) = \partial_{x,y}\phi^*(x, y, k)$$
$$B_z(x, y, k) = ik\phi^*(x, y, k).$$

The last term will not affect our calculation because it has no corresponding **E**-field.

For a dipole field exterior to all currents generating the field, the potential is

$$\phi^* = \frac{\vec{\mu} \cdot \vec{r}}{r^3}, \tag{18}$$

or in terms of components

$$\phi^* = \frac{\mu_x x + \mu_y y + \mu_z z}{[z^2 + \rho^2]^{\frac{3}{2}}}.$$

Where I have used $\rho = \sqrt{x^2 + y^2}$ for simplicity when appropriate. The Fourier transform of this configuration is

$$\phi^*(x, y, k) = 2 \left[(\mu_x x + \mu_y y) \frac{k}{\rho} K_1(k\rho) - i\mu_z k K_0(k\rho) \right]. \tag{19}$$

Where K_i is a modified bessel function.

For $k\rho \gg 1$, the following assymptotic expansion is valid:

$$K_i(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \left\{ 1 + \frac{4i^2 - 1}{8z} + \ldots \right\}$$

Therefore, one can simplify eq. (19):

$$\phi^*(x, y, k) = \left[\frac{\vec{\mu_\perp} \cdot \vec{\rho}}{\rho} - i\mu_z \right] \sqrt{\frac{2\pi k}{\rho}} e^{-k\rho} \tag{20}$$

We can now take the gradiant of this (in the x, y-plane) to find **B**.

$$\mathbf{B}(\rho, \theta, k) = \sqrt{\frac{2\pi k}{\rho}} e^{-k\rho} \left\{ \hat{\rho} \left[k(i\mu_z - \mu_\perp \cos\theta) + \frac{1}{2\rho}(i\mu_z - 3\mu_\perp \cos\theta) \right] + \frac{\vec{\mu_\perp}}{\rho} \right\}. \tag{21}$$

It is worth noting that the leading behavior in ρ for $k \gg 1/\rho$ is

$$\mathbf{B}(k) = \hat{\rho} k (i\mu_z - \mu_\perp \cos\theta) \sqrt{\frac{2\pi k}{\rho}} e^{-k\rho}. \tag{22}$$

6 Required $\mathbf{B}(k)$

We now estimate the required B-field to get significant photodisintegration due to the static fields, for the various field configurations that we have described. We use the approximate cross sections from Ref. [3]. The cross sections are given in units of the classical dipole sum rule or $\sum_d = 59.8\frac{NZ}{A}$ MeV-mb, and for almost all of the species listed, the integrated cross section is about $1\sum_d$ for single nucleon emission and an additional $(.1-.2)\sum_d$ for double nucleon emission. The cross sections generally reach a maximum around 20 MeV with a width of $8-12$ MeV. Clearly, the behavior is similar for all of these nuclei. Therefore, we will simply study an extremely simplified case. We will take $\sum_d = 60\frac{NZ}{A}$ and use N, Z, and A for iron. We will take the integrated cross section to be $1\sum_d$ centered at 20 MeV with a width of 8 MeV. We will ignore any variation in $\mathbf{B}(k)$ over this range and simply substitute the value at $20MeV$. All of these approximations should be correct to within 20% for iron and will not result in any qualitative changes in the result.

Using this data I calculated the magnetic field necessary to get a decay probability of $1-e^{-1}$ for an iron nucleus with a gamma factor of 10^{10} or an energy of about $5\times10^{20}eV$. The results can be found in table 2. These results can be modified for other values of gamma by multiplying k times $\frac{10^{10}}{\gamma}$. We find that k is approximately 100/cm. (Incidentally, this is close to the shorter wavelength part of the CMB, which also starts to be important at these energies. [8]) Entering this in equation (12) and restoring the $1/(c\sqrt{2\pi})$ we find that the necessary $B(k)$ is

$$\mathbf{B}(k)k = 1.7\times10^{7}\text{gauss}.$$

Substituting this into our results from table 1 we can make estimates for the required magnitude of the magnetic field for each of the field configurations listed. (Note: Discontinuity refers to the first derivative that changes abruptly on the scale of $1/k$.)

It is difficult to imagine objects which would contain fields of this magnitude. Since one would expect the magnetic field strengths to be continuous and to fall off over scales of at least a few km, we need magnetic fields of at least a 10^{14} gauss. For smoother decreases, larger objects, or motion that is not perpendicular to the magnetic field the prospects get worse. However, for higher energies or magnetic fields with repeated large variations the effect may become noticable.

Table 2: Required B-field for photodisintegration

Discontinuity	Required B field	Value
B	$\frac{B(k)k}{\sqrt{2}}$	1.2×10^7 gauss
B'	$\frac{B(k)k}{2} kb$	$8.5 \times 10^{13} \frac{\text{gauss}}{\text{km}} b$
B''	$\frac{B(k)k}{2\sqrt{3}} (kb)^2$	$4.9 \times 10^{20} \frac{\text{gauss}}{\text{km}^2} b^2$
B'''	$\frac{B(k)k}{4\sqrt{10}} (kb)^3$	$2.6 \times 10^{27} \frac{\text{gauss}}{\text{km}^3} b^3$
None	$\frac{B(k)k}{\sqrt{2}} \exp\left(\frac{(kb)^2}{4}\right)$	$1.2 \times 10^7 \exp\left(\frac{2.5\times10^{13} b^2}{\text{km}^2}\right)$ gauss

7 Accelerating Particles

All of the above treatment assumes that the particles are moving at a constant speed through the magnetic field, and that photodisintegration is caused by changes in the magnetic field. It is also possible that a particle is accelerated in the presence of a magnetic field, and that the changes in its perceived magnetic field due to this acceleration could cause photodisintegration. We discuss this possibility here.

If a particle feels a continuous proper acceleration a, (or equivalently, if it is pushed with a constant exterior force of $F = m_0 a$) then the relation between γ, x, and the proper time τ is as follows.

$$\gamma(\tau) = \cosh(a\tau + b)$$

$$x(\tau) = \frac{1}{a}\cosh(a\tau + b) + C$$

Here b and C are integration constants determined by the innitial conditions, and we have set the speed of light equal to 1.

Hyperbolic functions can be difficult to work with, however, as long as $\cosh(a\tau + b) \gg 1$ we can make the substitution $\cosh(a\tau + b) \approx \frac{1}{2}\exp(a\tau + b)$. Then the above equations simplify to

$$\gamma(\tau) = \gamma_0 \exp(a\tau) \tag{23}$$

$$x(\tau) = \frac{\gamma_0}{a}[\exp(a\tau) - 1] + x_0. \tag{24}$$

We would like to follow the same procedure as in sections 2 and 3 to find a power spectrum for the effective electromagnetic field that the particle sees. Most of this except for the Fourier transform will proceed as before. However in finding

$$\mathbf{B}(\omega) = \int_{-\infty}^{\infty} \mathbf{B}(\tau) e^{i\omega\tau} d\tau$$

a number of changes are in order. First, during acceleration the relationship between x and τ is now described by equation (24). Second, while it is reasonable to approximate a particle as always moving at a constant velocity, it is not reasonable to approximate it as always accelerating. Therefore, we must consider the particle's motion before, during, and after the acceleration. It may be tempting to simply assume (correctly) that most of the contribution to the Fourier transform will come from the acceleration, and therefore ignore the incoming and outgoing motion. However, to do so would introduce spurious contributions as we shall see. We will therefore assume that the particle comes in from $-\infty$ to point x_0 with γ_0, that it accelerates between x_0 and x_1 from γ_0 to γ_1, and that it finally leaves to ∞ with γ_1. The correct form for the Fourier transform, after changing integration variables is then

$$\begin{aligned}
\mathbf{B}(\omega) &= \int_{-\infty}^{x_0} \mathbf{B}(x) e^{i\frac{\omega}{\gamma_0}(x-x_0)} dx && (25)\\
&+ \int_{x_0}^{x_1} \mathbf{B}(x) [\frac{a}{\gamma_0}(x-\tilde{x})]^{i\frac{\omega}{a}} dx && (26)\\
&+ (\frac{\gamma_1}{\gamma_0})^{i\frac{\omega}{a}} \int_{x_0}^{\infty} \mathbf{B}(x) e^{i\frac{\omega}{\gamma_0}(x-x_1)} dx. && (27)
\end{aligned}$$

Where $\tilde{x} = x_0 - \frac{\gamma_0}{a} = x_1 - \frac{\gamma_1}{a}$.

If we assume that the magnetic field is constant except that it falls off slowly at $\pm\infty$ so that we can ignore the integration limits there, we find that

$$\mathbf{B}(\omega) = \left\{ \left(\frac{\gamma_1}{\gamma_0}\right)^{i\frac{\omega}{a}} \gamma_1 - \gamma_0 \right\} \left[\frac{1}{i\omega + a} - \frac{1}{i\omega} \right] = \left\{ \left(\frac{\gamma_1}{\gamma_0}\right)^{i\frac{\omega}{a}} \gamma_1 - \gamma_0 \right\} \frac{a}{\omega^2 - ia\omega}. \quad (28)$$

Where the first term in [ ] comes from the acceleration integral and the second term comes from the incoming and outgoing integrals. Notice that as mentioned before, the overall behavior in ω is softer than in either term alone. If we had integrated only over the acceleration region we would have essentially introduced a fictitious abrupt cutoff in the magnetic field. The incoming and outgoing motion cancels this effect. We can see that if the acceleration is much larger than ω the falloff with ω is fairly slow, and the problem becomes similar to an abrupt falloff in the B-field. As can be seen in table 2 this may be an achievable magnetic field in some extreme environments. However, such high accelerations require that the particle's energy change by nine orders of magnitude in a distance of a few microns. Thus, we expect much softer behavior, which would require much larger magnetic fields to achieve photodisintegration.

8 Conclusion

It apears unlikely that static magnetic fields would cause significant photodisintegration of nuclei, unless the fields are rapidly changing or the particles are accelerating, on scales of a few microns to a few millimeters. Not coincidentally, this is the same scale as the shorter wavelengths of the CMB [8] which cause attenuation of such high energy cosmic rays as they traverse the universe.

While this suggests that static magnetic fields are unlike to cause large energy losses in cosmic ray nuclei, the effect may become more significant at higher energies, or if the magnetic fields experience repeated variations.

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References

[1] For reviews, see, *e.g.*, P. Bhattacharjee and G. Sigl, Phys. Rept. 327,(2000), 109;J.W. Cronin, Ultra high energy cosmic rays. Nuclear Physics B, Proceedings Supplements,vol.97, (2001) 3; M. Nagano and A. A. Watson, Rev. Mod. Phys. 72, (2000)689; T.J. Weiler, hep-ph/0103023.

[2] K. Greisen, Phys. Rev. Lett. **16**, (1966) 748; G. T. Zatsepin and V. A. Kuzmin, Pisma Zh. Eksp. Teor. Fiz. **4**, (1966) 114.

[3] J.L. Puget, F.W. Stecker, J.H. Bredekamp, Astrophysical Journal, vol 205 (1976) 638.

[4] P. Blasi, R. I. Epstein, A. V. Olinto,Astrophys.J. 533 (2000) L123, astro-ph/9912240.

[5] J.D. Jackson, Classical Electrodynamics (J. Wiley and Sons, 1975), Section 15.4, pg. 719.

[6] X. Artru, G.B. Yodh, Phys. Letters 40B (1972) 43.

[7] Particle Data Group, Review of Particle Physics, European Physical Journal C, Volume 3, Number 1-4, (1998), Section 7, pg. 78.

[8] Particle Data Group, Review of Particle Physics, European Physical Journal C, Volume 3, Number 1-4, (1998), Section 19, pg. 127.